UNIT 5

LINEAR PROGRAMMING PROBLEMS (10 MARKS)

Topic Content:

5.1 Introduction, Basic terms in Linear Programming Problems.

5.2 Mathematical formulation of Linear Programming Problems.

MR SUDHIR S DESAI

5.3 Method of solving Linear Programming Problems (Two equations in two variables): Graphical (corner point) method, Simplex method.

Course Outcome: After completion of this course, students will be able to

CO5: Apply the linear programming problem concept to obtain optimal solution.

❖ Introduction.

In Mathematics, **linear programming** is a method of optimising operations with some constraints. The main objective of linear programming is to maximize or minimize the numerical value. It consists of linear functions which are subjected to the constraints in the form of linear equations or in the form of inequalities. Linear programming is considered an important technique that is used to find the optimum resource utilisation. The term "linear programming" consists of two words as linear and programming. The word "linear" defines the relationship between multiple variables with degree one. The word "programming" defines the process of selecting the best solution from various alternatives.

Linear Programming is widely used in Mathematics and some other fields such as economics, business, telecommunication, and manufacturing fields. In this article, let us discuss the definition of linear programming, its components, and different methods to solve linear programming problems.

❖ Formation of LPP:

STEP I: Define the decision variables.

STEP II: Construct the objective function which has to be optimized as a linear equation involving decision variables.

STEP III: Express every condition as a linear inequality involving decision variables.

NIVOLHAPUR

❖ **Example:**

1. **A company manufactures two types of toys A and B. Each type of toy A requires 2 minutes for cutting and 1 min for assembling. Each type of toy B requires 3 min for cutting and 4 minutes for assembling. There are 3 hrs. for cutting and 2 hrs. for assembling. On selling a toy of type A the company gets profit of Rs. 10 and that on toy B is Rs. 20. Formulate the LPP to maximize profit. [Summer-24] 2M**

Sol: Let us prepare first table of given data.

	Toy A	Toy B	Availability/Limitations
Cutting	2	1	3
Assembling	3	4	2
Profit	10	20	

STEP I: Consider company manufactures x_1 number of toys A & x_2 number of toys B

Total profit on selling toys A is $10x_1$

Total profit on selling toys B is $20x_2$

STEP II: Maximum profit $Z = 10x_1 + 20x_2$

STEP III: Subjected to

$$2x_1 + x_2 \leq 3$$
$$3x_1 + 4x_2 \leq 2$$
$$x_1, x_2 \geq 0$$

2. **A company manufactures two products P_1 & P_2. Product P_1 required 5 units & P_2 requires 6 units of electronic components. The supply of electronics components is limited to 600 units per day, labour supply is limited 160 man day. one unit of product P_1 requires 1 man day of labour and one unit of P_2 requires 2 man days of labour. Formulate the LPP to produce maximum profit. Profit earns is 50 Rs. From P_1 & 80 Rs. From one unit of P_2.**

Sol: Let us prepare first table of given data.

	Product P_1	Product P_2	Availability/Limitations
Electronic Components	5	6	600
Labour	1	2	160
Profit	50	80	

STEP I: Consider company manufactures x_1 number of product P_1 & x_2 number of product P_2

Total profit on selling product P_1 is $50x_1$

Total profit on selling product P_2 is $80x_2$

STEP II: Maximum profit $Z = 50x_1 + 80x_2$

STEP III: Subjected to

$$5x_1 + 6x_2 \leq 600$$
$$x_1 + 2x_2 \leq 160$$
$$x_1, x_2 \geq 0$$

NIT KOLHAPUR

3. A furniture dealer deals only two items, like table & chairs. He has to invest Rs. 10,000 |- & a space to store at most 60 pieces. A table cost him Rs. 500 & a chair cost him Rs. 200. He can sell all the items that he buys. He is getting a profit of Rs. 50 per table & Rs. 15 per chair. Formulate this problem as an LPP, so as to maximize the profit.

Sol: Let us prepare first table of given data.

	Table	Chairs	Availability/Limitations
Space			60
Cost	500	200	10,000
Profit	50	15	

STEP I: Consider furniture dealer manufactures x_1 number of tables & x_2 number of chairs.

Total profit on selling tables is $50x_1$

Total profit on selling chairs is $15x_2$

STEP II: Maximum profit $Z = 50x_1 + 15x_2$

STEP III: **Subjected to**

$$x_1 + x_2 \leq 60$$

$$500x_1 + 200x_2 \leq 10000$$

$$x_1, x_2 \geq 0$$

NIT KOLHAPUR

UNIT 5

LINEAR PROGRAMMING PROBLEMS (10 MARKS)

Topic Content:

5.1 Introduction, Basic terms in Linear Programming Problems.

MR SUDHIR S DESAI

5.2 Mathematical formulation of Linear Programming Problems.

5.3 Method of solving Linear Programming Problems (Two equations in two variables): Graphical (corner point) method, Simplex method.

Course Outcome: After completion of this course, students will be able to

CO4: Apply the linear programming problem concept to obtain optimal solution.

❖ Introduction.

In Mathematics, **linear programming** is a method of optimising operations with some constraints. The main objective of linear programming is to maximize or minimize the numerical value. It consists of linear functions which are subjected to the constraints in the form of linear equations or in the form of inequalities. Linear programming is considered an important technique that is used to find the optimum resource utilisation. The term "linear programming" consists of two words as linear and programming. The word "linear" defines the relationship between multiple variables with degree one. The word "programming" defines the process of selecting the best solution from various alternatives.

Linear Programming is widely used in Mathematics and some other fields such as economics, business, telecommunication, and manufacturing fields. In this article, let us discuss the definition of linear programming, its components, and different methods to solve linear programming problems.

❖ Definition:

It is a process of finding the optimal value (Maximum or Minimum) of a linear function (Objective function) of some variables, subject to linear constraints (equalities/inequalities)

❖ Graphical Method (Corner Method): 6M

The graphical method is used to optimize the two-variable linear programming. If the problem has two decision variables, a graphical method is the best method to find the optimal solution. In this method, the set of inequalities are subjected to constraints. Then the inequalities are plotted in the XY plane. Once, all the inequalities are plotted in the XY graph, the intersecting region will help to decide the feasible region. The feasible region will provide the optimal solution as well as explains what all values our model can take. Let us see an example here and understand the concept of linear programming in a better way.

STEP I: Write all inequality constraints in the form of equations.

STEP II: Plot these lines on a graph by identifying test points.

STEP III: Identify the feasible region.

STEP IV: Determine the coordinates of the corner points.

STEP V: Substitute each corner point in the objective function to find optimal solution.

AIT KOLHAPUR

❖ **Example:**

4. Solve the following LPP using graphical method :

Maximize $Z = 11x + 8y$

Subject to $x \le 4, y \le 6, x + y \le 6, x \ge 0, y \ge 0$. [summer 24 – 6M]

Sol: Given, Maximize $Z = 11x + 8y \ldots\ldots\ldots(1)$

Subject to $x \le 4, y \le 6, x + y \le 6, x \ge 0, y \ge 0.$

STEP I:

$x = 4, y = 6, x + y = 6$

STEP II:

$x = 4$ Is a straight line passing through $(4,0)$ & parallel to $y - axis$.

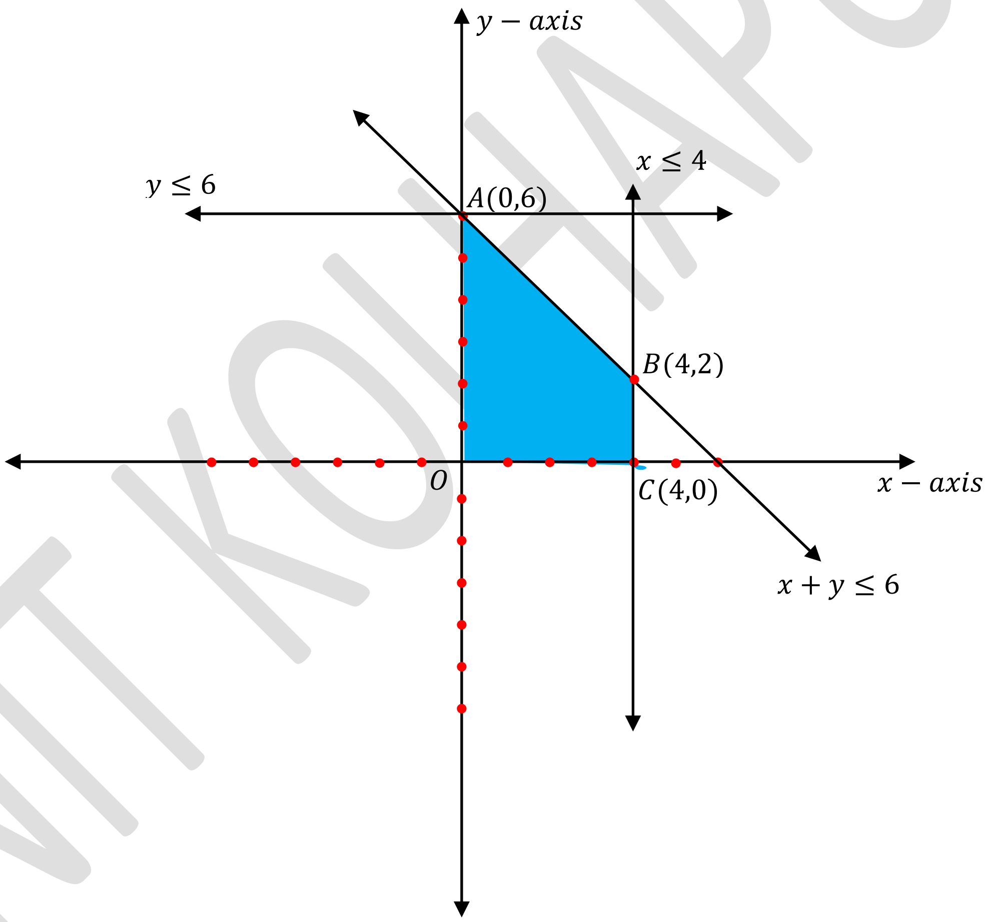
$y = 6$ Is a straight line passing through $(0,6)$ & parallel to $x - axis$.

$x = 0, y - axis$

$y = 0, x - axis$

For $x + y = 6$

x	0	6
y	6	0



To find intersection point B

Solving equations $x = 4 \ldots\ldots(1)$ & $x + y = 6 \ldots..(2)$

Put $x = 4$ in equation 2)

$\therefore 4 + y = 6 \rightarrow y = 6 - 4 \rightarrow \boxed{y = 2}$

The coordinates of point $B(4,2)$

STEP III: Feasible region is $OABC$

STEP IV: To find Z at corner points.

Corner Points	$Z = 11x + 8y$
$O(0,0)$	$\therefore Z = 11(0) + 8(0) = 0$
$A(0,6)$	$\therefore Z = 11(0) + 8(6) = 48$

$B(4,2)$	$\therefore Z = 11(4) + 8(2) = 60$
$C(4,0)$	$\therefore Z = 11(4) + 8(0) = 44$

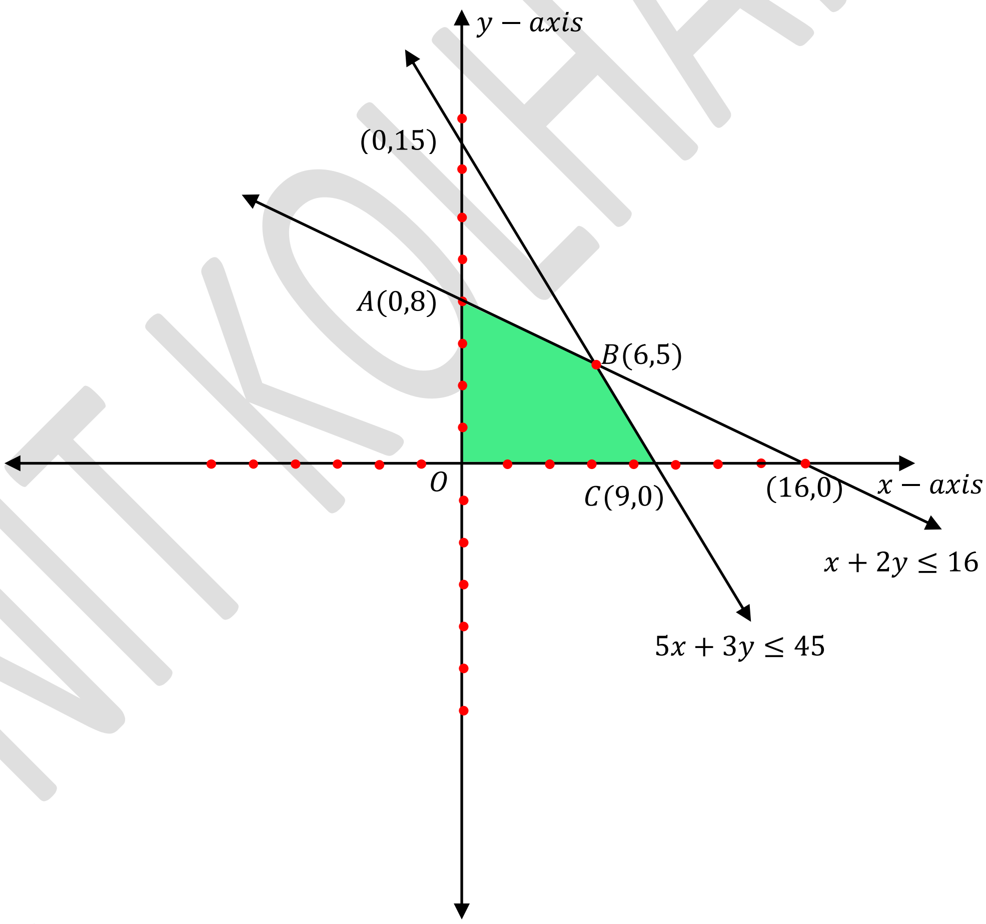
STEP V: Hence we find that Z is maximum at point $B(4,2)$ & Maximum value is $Z = 60$

5. **Solve the following Linear Programming Problem graphically to find optimal solution:**
Maximize $Z = 2x + 5y$
Subject to $x + 2y \leq 16$, $5x + 3y \leq 45$, $x \geq 0$, $y \geq 0$. [summer 23 - 6M]
Sol: Given, Maximize $Z = 2x + 5y$ ……….(1)
Subject to $x + 2y \leq 16$, $5x + 3y \leq 45$, $x \geq 0$, $y \geq 0$.
STEP I:
$x + 2y = 16$, $5x + 3y = 45$, $x = 0$, $y = 0$.
STEP II:
$x = 0$, $y - axis$
$y = 0$, $x - axis$
For $x + 2y = 16$

x	0	16
y	8	0

For $5x + 3y = 45$

x	0	9
y	15	0



To find intersection point B

Solving equations $x + 2y = 16 \rightarrow x = 16 - 2y$ ……..(1) & $5x + 3y = 45$ …..(2)

Put $x = 16 - 2y$ in equation 2)
$\therefore 5(16 - 2y) + 3y = 45$
$\therefore 80 - 10y + 3y = 45$

$\therefore -7y = 45 - 80 \rightarrow -7y = -35 \rightarrow y = \frac{-35}{-7}$ $\boxed{\therefore y = 5}$

Put $y = 5$ in equation 1)

$x = 16 - 2(5) \rightarrow x = 16 - 10$ $\therefore \boxed{x = 6}$

The coordinates of point $B(6,5)$

STEP III: Feasible region is $OABC$

STEP IV: To find Z at corner points.

Corner Points	$Z = 2x + 5y$
$O(0,0)$	$\therefore Z = 2(0) + 5(0) = 0$
$A(0,8)$	$\therefore Z = 2(0) + 5(8) = 40$
$B(6,5)$	$\therefore Z = 2(6) + 5(5) = 37$
$C(9,0)$	$\therefore Z = 2(9) + 5(0) = 18$

STEP V: Hence we find that Z is maximum at point $A(0,8)$ & Maximum value is $Z = 40$

6. Solve the following Linear Programming Problem graphically to find optimal solution:

Maximize $Z = 5x + 3y$

Subject to $3x + 5y \leq 15$, $5x + 2y \leq 10$, $x \geq 0$, $y \geq 0$. [winter 23 - 6M]

Sol: Given, Maximize $Z = 5x + 3y \dots\dots\dots(1)$

Subject to $3x + 5y \leq 15$, $5x + 2y \leq 10$, $x \geq 0$, $y \geq 0$.

STEP I:

$3x + 5y = 15$, $5x + 2y = 10$, $x = 0$, $y = 0$.

STEP II:

$x = 0$, $y - axis$

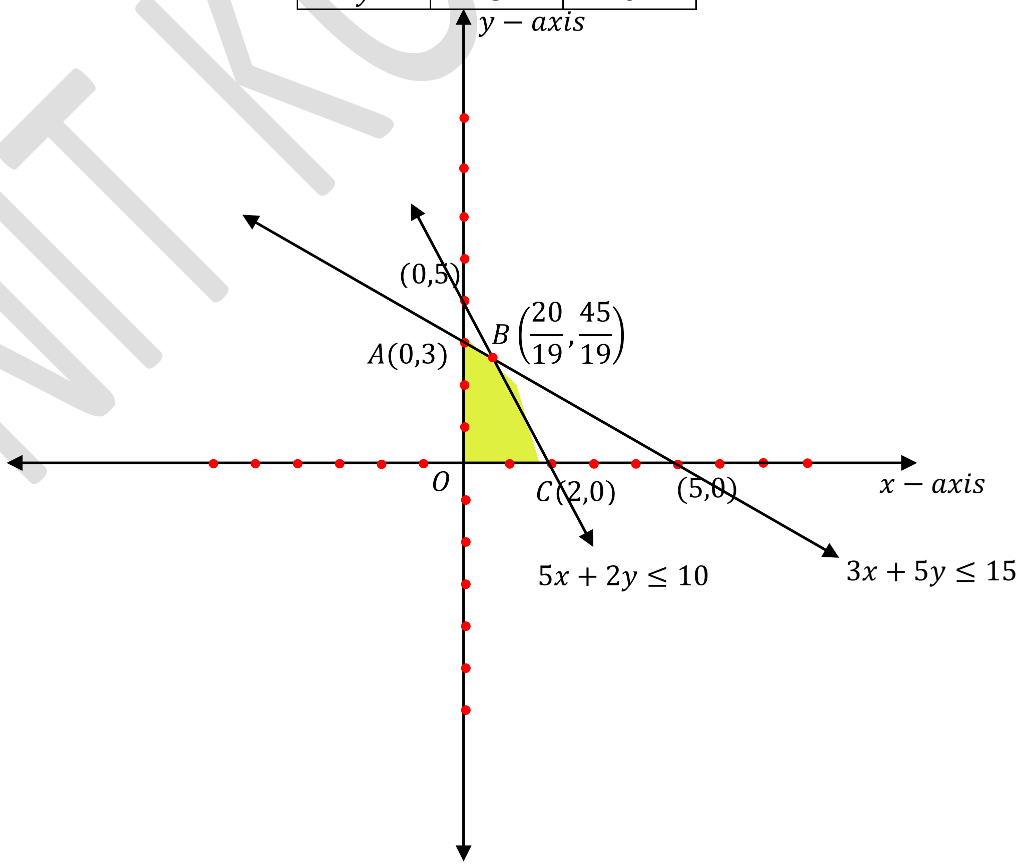
$y = 0$, $x - axis$

For $3x + 5y = 15$

x	0	5
y	3	0

For $5x + 2y = 10$

x	0	2
y	5	0



To find intersection point B

Solving equations $3x + 5y = 15 \rightarrow x = \frac{15-5y}{3} \rightarrow x = 5 - \frac{5}{3}y \ldots\ldots(1)$ & $5x + 2y = 10 \ldots\ldots(2)$

Put $x = 5 - \frac{5}{3}y$ in equation 2)

$\therefore 5\left(5 - \frac{5}{3}y\right) + 2y = 10$

$\therefore 25 - \frac{25}{3}y + 2y = 10$

$\therefore -\frac{19}{3}y = 10 - 25 \rightarrow -\frac{19}{3}y = -15 \rightarrow y = -15 \times -\frac{3}{19}$ $\boxed{\therefore y = \frac{45}{19}}$

Put $y = \frac{45}{19}$ in equation 1)

$\therefore x = 5 - \frac{5}{3} \times \frac{45}{19}$ $\therefore$ $\boxed{x = \frac{20}{19}}$

The coordinates of point $B\left(\frac{20}{19}, \frac{45}{19}\right)$

STEP III: Feasible region is $OABC$

STEP IV: To find Z at corner points.

Corner Points	$Z = 5x + 3y$
$O(0,0)$	$\therefore Z = 5(0) + 3(0) = 0$
$A(0,3)$	$\therefore Z = 5(0) + 3(3) = 9$
$B\left(\frac{20}{19}, \frac{45}{19}\right)$	$\therefore Z = 5\left(\frac{20}{19}\right) + 3\left(\frac{45}{19}\right) = 12.3684$
$C(2,0)$	$\therefore Z = 2(2) + 5(0) = 4$

STEP V: Hence we find that Z is maximum at point $B\left(\frac{20}{19}, \frac{45}{19}\right)$ & Maximum value is $Z = 12.3684$

- **Homework:**

1. **Calculate the maximal and minimal value of $z = 5x + 3y$ for the following constraints graphically.**

   $x + 2y \leq 14, 3x - y \geq 0, x - y \leq 2, x \geq 0, y \geq 0$

   Ans: Max $z = 42$ at $(6,4)$ & Min $z = -14$ at $(-1,-3)$

2. **Solve the following Linear Programming Problem graphically to find optimal solution:**

   Maximize $Z = 2x_1 + 5x_2$

   Subject to $x_1 + 4x_2 \leq 2, 3x_1 + x_2 \leq 21, x_1 + x_2 \leq 9, x_1 \geq 0, x_2 \geq 0$. Ans: Max $z = 33$ at $(4,5)$

3. **Solve the following Linear Programming Problem graphically to find optimal solution:**

   Minimize $Z = 5x_1 + 4x_2$

   Subject to $4x_1 + 4x_2 \geq 40, 2x_1 + 3x_2 \geq 90, x_1 \geq 0, x_2 \geq 0$. Ans: Min $z = 127$ at $(3,28)$

4. **Solve the following Linear Programming Problem graphically to find optimal solution:**

   Maximize $Z = 2x_1 + 3x_2$

   Subject to $x_1 + x_2 \leq 30,\ x_1 \leq 20,\ x_2 \leq 12, x_1 \geq 0, x_2 \geq 0$.

NIT KOLHAPUR

UNIT 5

LINEAR PROGRAMMING PROBLEMS (10 MARKS)

Topic Content:

5.1 Introduction, Basic terms in Linear Programming Problems.

MR SUDHIR S DESAI

5.2 Mathematical formulation of Linear Programming Problems.

5.3 Method of solving Linear Programming Problems (Two equations in two variables): Graphical (corner point) method, **Simplex method.**

Course Outcome: After completion of this course, students will be able to

CO5: Apply the linear programming problem concept to obtain optimal solution.

❖ **Introduction.**

Simplex method also called simplex technique or simplex algorithm was developed by G.B. Dantzig, An American mathematician. Simplex method is suitable for solving linear programming problems with a large number of variables. The method through an iterative process progressively approaches and ultimately reaches to the maximum or minimum values of the objective function.

❖ **Procedure to find Maximum value using Simplex Method:**

STEP 1) Write objective function & constraints by adding basic (slack) variables $(x_3 \& x_4)$

STEP 2) Construct an initial simplex table as follows.

		C_j	Coefficient of variables in Z				Min Ratio $\frac{b}{x_i}$
C_b	X_b	b	x_1	x_2	x_3	x_4	
Coeff of x_3 in z	x_3	Const. on RHS 1	Coefficient of variables in equation 1				
Coeff of x_4 in z	x_4	Const. on RHS 2	Coefficient of variables in equation 2				
		$\sum[C_b \times x_i] = Z_j$					
		$Z_j - C_j$					

STEP 3) Find the entering variable whose coefficient in $Z_j - C_j$ row is most negative

STEP 4) Find the exit variable i.e. the basic variable in the row where the min ratio is small as possible. (non negative).

STEP 5) Find the Key Element at the intersection of the entering variable column the exit variable row.

STEP 6) Make key element one by dividing the row by Key element

STEP 7) Now make element above or below Key element zero & construct new table

STEP 8) Repeat the steps form 2) to 7) until you get all $Z_j - C_j \geq 0$.

STEP 9) Find value of $x_1 \& x_2$ from final table & put in Z to find optimal solution.

NITKOLHAPUR

❖ **Example:**

7. Solve the following Linear programming problem using Simplex method to find optimal solution:

Maximize $Z = 3x_1 + 4x_2$

Subject to $x_1 + x_2 \leq 450$, $2x_1 + x_2 \leq 600, x_1 \geq 0, x_2 \geq 0$. [winter 23 – 6M]

Sol: Consider,

$Z = 3x_1 + 4x_2 + 0x_3 + 0x_4$

$x_1 + x_2 + x_3 + 0x_4 = 450$

$2x_1 + x_2 + 0x_3 + x_4 = 600$

Table 1:

Key Element

		C_j	3	4	0	0	Min Ratio $\frac{b}{x_2}$
C_b	X_b	b	x_1	x_2	x_3	x_4	
0	x_3	450	1	1	1	0	$\frac{450}{1} = 450$
0	x_4	600	2	1	0	1	$\frac{600}{1} = 600$
		$\sum[C_b \times x_i] = Z_j$	0	0	0	0	
		$Z_j - C_j$	-3	-4	0	0	

Exit Variable

Enter Variable

Here, key element is already one

To make element below key element equal to zero.

Old line – New line

600 – 450	150
2 – 1	1
1 – 1	0
0 – 1	-1
1 – 0	1

Table 2:

		C_j	3	4	0	0
C_b	X_b	b	x_1	x_2	x_3	x_4
4	x_2	450	1	1	1	0
0	x_4	150	1	0	-1	1
		$\sum[C_b \times x_i] = Z_j$	4	4	4	0
		$Z_j - C_j$	1	0	4	0

Here all $Z_j - C_j \geq 0$ $\therefore$ $\boxed{x_1 = 0, x_2 = 450}$

The maximum value of $Z = 3(0) + 4(450)$

$\boxed{\therefore Z = 1800}$

NIT KOLHAPUR

8. Solve the following Linear programming problem using Simplex method to find optimal solution:

Maximize $Z = 12x_1 + 16x_2$

Subject to $10x_1 + 20x_2 \leq 120, 8x_1 + 8x_2 \leq 80, x_1 \geq 0, x_2 \geq 0.$ **[summer 23 – 6M]**

Sol: Consider,

$Z = 12x_1 + 16x_2 + 0x_3 + 0x_4$

$10x_1 + 20x_2 + x_3 + 0x_4 = 120$

$8x_1 + 8x_2 + 0x_3 + x_4 = 80$

Table 1:

Key Element

		C_j	12	16	0	0	Min Ratio
C_b	X_b	b	x_1	x_2	x_3	x_4	$\frac{b}{x_2}$
0	x_3	120	10	**20**	1	0	$\frac{120}{20} = 6$ → Exit Variable
0	x_4	80	8	8	0	1	$\frac{80}{8} = 10$
		$\sum[C_b \times x_i] = Z_j$	0	0	0	0	
		$Z_j - C_j$	-12	**-16**	0	0	

Enter Variable

Here, key element is 20

So make it one by dividing each element by 20

To make element below key element equal to zero.

Old line – 8 x New line

80 – 8 x 6	32
$8 - 8 \times \frac{1}{2}$	4
8 – 8 x 1	0
$0 - 8 \times \frac{1}{20}$	$-\frac{2}{5}$
1 – 8 x 0	1

Table 2:

Key Element

		C_j	12	16	0	0	Min Ratio
C_b	X_b	b	x_1	x_2	x_3	x_4	$\frac{b}{x_1}$
16	x_2	6	$\frac{1}{2}$	1	$\frac{1}{20}$	0	$\frac{6}{1/2} = 12$
0	x_4	32	**4**	0	$-\frac{2}{5}$	1	$\frac{32}{4} = 8$ → Exit Variable
		$\sum[C_b \times x_i] = Z_j$	8	16	$\frac{4}{5}$	0	
		$Z_j - C_j$	**-4**	0	$\frac{4}{5}$	0	

Enter Variable

Here, key element is 4

So, make it one by dividing each element by 4

To make element above key element equal to zero.

NIT KOLHAPUR

Old line – $\frac{1}{2}$ x New line

$6-\frac{1}{2}\text{x}\,8$	2
$\frac{1}{2}-\frac{1}{2}\text{x}\,1$	0
$1-\frac{1}{2}\text{x}\,0$	1
$\frac{1}{20}-\frac{1}{2}\text{x}-\frac{1}{10}$	$\frac{1}{10}$
$0-\frac{1}{2}\text{x}\,\frac{1}{4}$	$\frac{-1}{8}$

Table 3:

		C_j	12	16	0	0
C_b	X_b	b	x_1	x_2	x_3	x_4
16	x_2	2	0	1	$\frac{1}{10}$	$\frac{-1}{8}$
12	x_1	8	1	0	$-\frac{1}{10}$	$\frac{1}{4}$
		$\sum[C_b \times x_i] = Z_j$	12	16	$\frac{2}{5}$	1
		$Z_j - C_j$	0	0	$\frac{2}{5}$	1

Here all $Z_j - C_j \geq 0$ $\therefore$ $\boxed{x_1 = 8, x_2 = 2}$

The maximum value of $Z = 12(8) + 16(2)$

$\boxed{\therefore Z = 128}$

9. Solve the following Linear programming problem using Simplex method to find optimal solution:
Maximize $Z = 5x_1 + 3x_2$
Subject to $3x_1 + 5x_2 \leq 15, 5x_1 + 2x_2 \leq 10, x_1 \geq 0, x_2 \geq 0$.

Sol: Consider,

$Z = 5x_1 + 3x_2 + 0x_3 + 0x_4$

$3x_1 + 5x_2 + x_3 + 0x_4 = 15$

$5x_1 + 2x_2 + 0x_3 + x_4 = 10$

Key Element

Table 1:

		C_j	5	3	0	0	Min Ratio $\frac{b}{x_1}$
C_b	X_b	b	x_1	x_2	x_3	x_4	
0	x_3	15	3	5	1	0	$\frac{15}{3} = 5$
0	x_4	10	5	2	0	1	$\frac{10}{5} = 2$
		$\sum[C_b \times x_i] = Z_j$	0	0	0	0	
		$Z_j - C_j$	- 5	-3	0	0	

Exit Variable

Enter Variable

Here, key element is 5

So make it one by dividing each element by 5

To make element above key element equal to zero.

Old line – 3 x New line

$15 - 3 \times 2$	9
$3 - 3 \times 1$	0
$5 - 3 \times \frac{2}{5}$	$\frac{19}{5}$
$1 - 3 \times 0$	1
$0 - 3 \times \frac{1}{5}$	$\frac{-3}{5}$

Table 2:

Key Element

		C_j	5	3	0	0	Min Ratio $\frac{b}{x_2}$
C_b	X_b	b	x_1	x_2	x_3	x_4	
0	x_3	9	0	$\frac{19}{5}$	1	$-\frac{3}{5}$	$9 \times \frac{5}{19} = \frac{45}{19} = 2.37$
5	x_1	2	1	$\frac{2}{5}$	0	$\frac{1}{5}$	$2 \times \frac{5}{2} = 5$
		$\sum [C_b \times x_i] = Z_j$	5	2	0	1	
		$Z_j - C_j$	0	-1	0	1	

Exit Variable

Enter Variable

Here, key element is $\frac{19}{5}$

So make it one by multiplying each element by $\frac{5}{19}$

To make element below key element equal to zero.

Old line – $\frac{2}{5}$ x New line

$2 - \frac{2}{5} \times \frac{45}{19}$	$\frac{20}{19}$
$1 - \frac{2}{5} \times 0$	1
$\frac{2}{5} - \frac{2}{5} \times 1$	0
$0 - \frac{2}{5} \times \frac{5}{19}$	$-\frac{2}{19}$
$\frac{1}{5} - \frac{2}{5} \times \frac{-3}{19}$	$\frac{5}{19}$

Table 3:

		C_j	5	3	0	0
C_b	X_b	b	x_1	x_2	x_3	x_4
3	x_2	$\frac{45}{19}$	0	1	$\frac{5}{19}$	$-\frac{3}{19}$
5	x_1	$\frac{20}{19}$	1	0	$-\frac{2}{19}$	$\frac{5}{19}$
		$\sum [C_b \times x_i] = Z_j$	5	3	$\frac{5}{19}$	$\frac{16}{19}$
		$Z_j - C_j$	0	0	$\frac{5}{19}$	$\frac{16}{19}$

NIT KOLHAPUR

Here all $Z_j - C_j \geq 0$ $\quad \therefore \boxed{x_1 = \frac{20}{19}, x_2 = \frac{45}{19}}$

The maximum value of $Z = 5\left(\frac{20}{19}\right) + 3\left(\frac{45}{19}\right)$

$\boxed{\therefore Z = 12.3684}$

- **Homework:**

1. **Find solution using Simplex method.** **(not in syllabus but you can try)**

   Maximize $Z = 30x_1 + 40x_2$

   Subject to $3x_1 + 2x_2 \leq 600, 3x_1 + 5x_2 \leq 800, 5x_1 + 6x_2 \leq 1100\ x_1 \geq 0, x_2 \geq 0.$

   [Ans: $x_1 = 100, x_2 = 100$, **Max** $z = 7000$**]**

2. **Find solution using Simplex method.**

   Maximize $Z = 40x_1 + 30x_2$

   Subject to $x_1 + x_2 \leq 12, 2x_1 + x_2 \leq 16,\ x_1 \geq 0, x_2 \geq 0.$

   [Ans: $x_1 = 4, x_2 = 8$, **Max** $z = 400$**]**

3. **Find solution using Simplex method.**

   Maximize $Z = 3x_1 + 2x_2$

   Subject to $x_1 + x_2 \leq 4, x_1 + 3x_2 \leq 6, x_1 \geq 0, x_2 \geq 0.$

   [Ans: $x_1 = 3, x_2 = 1$, **Max** $z = 13$**]**

NIT KOLHAPUR

UNIT 5

LINEAR PROGRAMMING PROBLEMS (10 MARKS)

Topic Content:

5.1 Introduction, Basic terms in Linear Programming Problems.

MR SUDHIR S DESAI

5.2 Mathematical formulation of Linear Programming Problems.

5.3 Method of solving Linear Programming Problems (Two equations in two variables): Graphical (corner point) method, **Simplex method Big M Method.**

Course Outcome: After completion of this course, students will be able to

CO5: Apply the linear programming problem concept to obtain optimal solution.

- **Procedure to find Maximum value using Simplex Method:**

STEP 1) Write minimization problem as $Maximize\ z' = -Min\ z$

STEP 2) Write objective function & constraints by subtracting surplus variables $(x_3 \& x_4)$ **and adding artificial variables** $(A_1 \& A_2)$

STEP 3) Construct an initial simplex table as follows.

		C_j	Coefficient of variables in Z						Min Ratio $\frac{b}{x_i}$
C_b	X_b	b	x_1	x_2	x_3	x_4	A_1	A_2	
Coeff of A_1 in z	A_1	Const. on RHS 1	Coefficient of variables in equation 1						
Coeff of A_2 in z	A_2	Const. on RHS 2	Coefficient of variables in equation 2						
$\sum[C_b \times x_i] = Z_j$									
$Z_j - C_j$									

STEP 4) Find the entering variable whose coefficient in $Z_j - C_j$ **row is most negative**

STEP 5) Find the exit variable i.e. the basic variable in the row where the min ratio is small as possible. (non negative).

STEP 6) Find the Key Element at the intersection of the entering variable column the exit variable row.

STEP 7) Make key element one by dividing the row by Key element

STEP 8) Now make element above or below Key element zero & construct new table by eliminating artificial variable

STEP 9) Repeat the steps form 2) to 7) until you get all $Z_j - C_j \geq 0$.

STEP 10) Find value of $x_1 \& x_2$ **from final table & put in Z to find optimal solution.**

NITKOLHAPUR

❖ **Example:**

10. Solve the following Linear programming problem using Simplex method to find optimal solution:

Minimize $Z = x_1 + x_2$

Subject to $2x_1 + x_2 \geq 4, x_1 + 7x_2 \geq 7, x_1 \geq 0, x_2 \geq 0.$

Sol: Consider, $Max\ Z' = -Min\ Z$

$Max\ Z' = -x_1 - x_2 + 0x_3 + 0x_4 - MA_1 - MA_2$

$2x_1 + x_2 - x_3 + 0x_4 + A_1 + 0A_2 = 4$

$x_1 + 7x_2 + 0x_3 - x_4 + 0A_1 + A_2 = 7$

Key Element

Table 1:

		C_j	-1	-1	0	0	$-M$	$-M$	Min Ratio $\frac{b}{x_2}$
C_b	X_b	b	x_1	x_2	x_3	x_4	A_1	A_2	
$-M$	A_1	4	2	1	-1	0	1	0	$\frac{4}{1} = 4$
$-M$	A_2	7	1	7	0	-1	0	1	$\frac{7}{7} = 1$ → Exit Variable
$\sum[C_b \times x_i] = Z_j$			$-3M$	$-8M$	M	M	$-M$	$-M$	
$Z_j - C_j$			$-3M + 1$	$-8M + 1$	M	M	0	0	

↑ Enter Variable

Here, key element is 7

To make it one divide each element of that row by 7

To make element above key element equal to zero.

Old line – New line

$4 - 1$	3
$2 - \frac{1}{7}$	$\frac{13}{7}$
$1 - 1$	0
$-1 - 0$	-1
$0 - \frac{-1}{7}$	$\frac{1}{7}$
$1 - 0$	1

Key Element

Table 2:

		C_j	-1	-1	0	0	$-M$	Min Ratio $\frac{b}{x_1}$
C_b	X_b	b	x_1	x_2	x_3	x_4	A_1	
$-M$	A_1	3	$\frac{13}{7}$	0	-1	$\frac{1}{7}$	1	$3 \times \frac{7}{13} = 1.62$ → Exit Variable
-1	x_2	1	$\frac{1}{7}$	1	0	$-\frac{1}{7}$	0	$1 \times \frac{7}{1} = 7$
$\sum[C_b \times x_i] = Z_j$			$\frac{-13M - 1}{7}$	1	M	$\frac{-M + 1}{7}$	$-M$	

NIT KOLHAPUR

$\boldsymbol{Z_j - C_j}$	$\frac{-13M-1}{7}+1$	0	M	$\frac{-M+1}{7}$	0

Enter Variable

Here, key element is $\frac{13}{7}$

To make it one multiply each element of that row by $\frac{7}{13}$

To make element above key element equal to zero.

Old line – $\frac{1}{7}$ **× New line**

$1-\frac{1}{7}\times\frac{21}{13}$	$\frac{10}{13}$
$\frac{1}{7}-\frac{1}{7}\times 1$	0
$1-\frac{1}{7}\times 0$	1
$0-\frac{1}{7}\times\frac{-7}{13}$	$\frac{1}{13}$
$\frac{-1}{7}-\frac{1}{7}\times\frac{1}{13}$	$-\frac{2}{13}$

<u>**Table 3:**</u>

		C_j	-1	-1	0	0
C_b	X_b	b	x_1	x_2	x_3	x_4
-1	x_1	$\frac{21}{13}$	1	0	$\frac{-7}{13}$	$\frac{1}{13}$
-1	x_2	$\frac{10}{13}$	0	1	$\frac{1}{13}$	$-\frac{2}{13}$
$\sum[C_b \times x_i] = Z_j$			-1	-1	$\frac{6}{13}$	$\frac{1}{13}$
$Z_j - C_j$			0	0	$\frac{6}{13}$	$\frac{1}{13}$

Here all $\boldsymbol{Z_j - C_j \geq 0}$ $\therefore$ $\boxed{x_1 = \frac{21}{13}, x_2 = \frac{21}{13}}$

The minimum value of $Z = \frac{21}{13} + \frac{10}{13}$

$$\boxed{\therefore Z = \frac{31}{13} = 2.3846}$$

NIT KOLHAPUR

11. Solve the following Linear programming problem using Simplex method to find optimal solution:

Minimize $Z = 4x_1 + 6x_2$

Subject to $x_1 + x_2 \geq 8, 6x_1 + x_2 \geq 12, x_1 \geq 0, x_2 \geq 0.$

Sol: Consider, $Max\ Z' = -Min\ Z$

$Max\ Z' = -4x_1 - 6x_2 + 0x_3 + 0x_4 - MA_1 - MA_2$

$x_1 + x_2 - x_3 + 0x_4 + A_1 + 0A_2 = 8$

$6x_1 + x_2 + 0x_3 - x_4 + 0A_1 + A_2 = 12$

Key Element

Table 1:

		C_j	-4	-6	0	0	$-M$	$-M$	Min Ratio $\frac{b}{x_1}$
C_b	X_b	b	x_1	x_2	x_3	x_4	A_1	A_2	
$-M$	A_1	8	1	1	-1	0	1	0	$\frac{8}{1} = 8$
$-M$	A_2	12	6	1	0	-1	0	1	$\frac{12}{6} = 2$
$\sum[C_b \times x_i] = Z_j$			$-7M$	$-2M$	M	M	$-M$	$-M$	
$Z_j - C_j$			$-7M + 4$	$-2M + 1$	M	M	0	0	

Exit Variable

Enter Variable

Here, key element is 6

To make it one divide each element of that row by 6

To make element above key element equal to zero.

Old line – New line

$8 - 2$	6
$1 - 1$	0
$1 - \frac{1}{6}$	$\frac{5}{6}$
$-1 - 0$	-1
$0 - \frac{-1}{6}$	$\frac{1}{6}$
$1 - 0$	1

Key Element

Table 2:

		C_j	-4	-6	0	0	$-M$	Min Ratio $\frac{b}{x_2}$
C_b	X_b	b	x_1	x_2	x_3	x_4	A_1	
$-M$	A_1	6	0	$\frac{5}{6}$	-1	$\frac{1}{6}$	1	$6 \times \frac{6}{5} = 7.2$
-4	x_1	2	1	$\frac{1}{6}$	0	$-\frac{1}{6}$	0	$2 \times \frac{6}{1} = 12$
$\sum[C_b \times x_i] = Z_j$			-4	$\frac{-5M - 4}{6}$	M	$\frac{-M + 4}{6}$	$-M$	

Exit Variable

NIT KOLHAPUR

$Z_j - C_j$	0	$\frac{-5M-4}{6}+6$	M	$\frac{-M+4}{6}$	0

↑ Enter Variable

Here, key element is $\frac{5}{6}$

To make it one multiply each element of that row by $\frac{6}{5}$

To make element below key element equal to zero.

Old line – $\frac{1}{6}$ $\times$ **New line**

$2-\frac{1}{6}\times\frac{36}{5}$	$\frac{4}{5}$
$1-\frac{1}{6}\times 0$	1
$\frac{1}{6}-\frac{1}{6}\times 1$	0
$0-\frac{1}{6}\times\frac{-6}{5}$	$\frac{1}{5}$
$-\frac{1}{6}-\frac{1}{6}\times\frac{1}{5}$	$-\frac{1}{5}$

Table 3:

		C_j	-4	-6	0	0	Min Ratio
C_b	X_b	b	x_1	x_2	x_3	x_4	$\frac{b}{x_4}$
-6	x_2	$\frac{36}{5}$	0	1	$\frac{-6}{5}$	$\frac{1}{5}$	$\frac{36}{5}\times\frac{5}{1}=36$ → Exit Variable
-4	x_1	$\frac{4}{5}$	1	0	$\frac{1}{5}$	$-\frac{1}{5}$	$\frac{4}{5}\times\frac{-5}{1}=-4$
$\sum[C_b\times x_i]=Z_j$			-4	-6	$\frac{32}{5}$	$-\frac{2}{5}$	
$Z_j - C_j$			0	0	$\frac{32}{5}$	$-\frac{2}{5}$	

↑ Enter Variable

Here, key element is $\frac{1}{5}$

To make it one multiply each element of that row by $\frac{5}{1}$

To make element below key element equal to zero.

Old line + $\frac{1}{5}$ $\times$ **New line**

$\frac{4}{5}+\frac{1}{5}\times 36$	8
$1+\frac{1}{5}\times 0$	1
$0+\frac{1}{5}\times 5$	1
$\frac{1}{5}+\frac{1}{5}\times -6$	-1
$-\frac{1}{5}+\frac{1}{5}\times 1$	0

NIT KOLHAPUR

Table 4:

		C_j	-4	-6	0	0
C_b	X_b	b	x_1	x_2	x_3	x_4
0	x_4	36	0	5	-6	1
-4	x_1	8	1	1	-1	0
$\sum[C_b \times x_i] = Z_j$			-4	-4	4	0
		$Z_j - C_j$	0	2	4	0

Here all $Z_j - C_j \geq 0$ $\quad\therefore \boxed{x_1 = 8, x_2 = 0}$

The minimum value of $Z = 4(8) + 6(0)$

$\boxed{\therefore Z = 32}$

NIT KOLHAPUR